| | COOMET RECOMMENDATION<br><br>**Calibration of measuring instruments. Algorithms of processing measurement results and uncertainty evaluation** | **COOMET R/GM/32:2017**<br><br>420/RU-a/08 |
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Contents

## 1 Field of application

The recommendations given apply to methods of calibration of measurement standard and measuring means. The recommendations set up general provisions, applied terms, definitions, methods and algorithms of measurement result evaluation in calibration and their uncertainty. The recommendations can be used for stating calibration and measurement capabilities (CMC) of National Metrology Institutes (NMI).

## 2 Normative references

In the present recommendations there are references to the following normative documents:

- ISO/IEC 17025 General requirements for the competence of testing and calibration laboratories.

## 3 Terms, definitions, accepted abbreviations and symbols

3.1 Terms and definitions

In the present recommendations the following terms are applied with corresponding definitions:

**Calibration of measuring instruments** is a group of operations determining the relationship between the value obtained with the help of a given measuring instrument and a corresponding quantity value determined with the help of a measurement standard with the aim to determine metrological characteristics of this measuring instrument.

**Metrological characteristic of a measuring instrument** is the characteristic of one of the properties of the measuring instrument, which influences the measurement result.

**Calibration characteristic of a measuring instrument** is the relationship between the value of a measurand and indications of a measuring instrument, which can be represented with a function, table or diagram accompanied by a corresponding uncertainty.

*Note:* In the document given two methods of presenting the *calibration characteristic of a measuring instrument* are considered, the first one in the form of MI indications dependence on measurand values, $y=f(x)$, and the second one in the form of the inverse function $x=f^{-1}(y)$. In the context it is always explained what form of presentation takes place.

**Measurement uncertainty** is the non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used.

**Standard uncertainty** is the uncertainty expressed in the form of a standard deviation.

**Evaluation of the measurement uncertainty according to Type A** is the evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions.

**Evaluation of the measurement uncertainty according to Type B** is the evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty.

**Combined standard uncertainty** is the standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model.

**Expanded uncertainty** is the product of a combined standard measurement uncertainty and a factor larger than the number one.

**Uncertainty budget** is the statement of a measurement uncertainty, of the components of that measurement uncertainty, and of their calculation and combination.

*Note*: An uncertainty budget should include the measurement model, estimates, and measurement uncertainties associated with the quantities in the measurement model, covariances, type of applied probability density functions, degrees of freedom, type of evaluation of measurement uncertainty, and any coverage factor.

**Measurement model** is the mathematical relationship between quantities in a concrete measurement problem and expressed in the form of the quantity equation.

**Measurement function** is the function of quantities, the value of which, when calculated using known quantity values for the input quantities in a measurement model, is a measured quantity value of the output quantity in the measurement model.

**Input quantity in a measurement model** is the quantity that must be measured, or a quantity, the value of which can be otherwise obtained, in order to calculate a measured quantity value of a measurand.

**Output quantity** is the quantity, the measured value of which is obtained using values of input quantities in a measurement model.

**Systematic measurement error** is the component of measurement error that in replicate measurements remains constant or varies in a predictable manner.

*Note*: Systematic error of a given measuring instrument, as a rule, will differ from the systematic error of another measuring instrument specimen of the same type, hence for a group of measuring instruments of the same type the systematic can be considered as a random error.

**Correction** is the quantity value introduced into indication with the aim to exclude components of the systematic error.

*Примечание*. Знак поправки противоположен знаку погрешности. Поправку, прибавляемую к номинальному значению меры, называют ***поправкой к значению меры***; поправку, вводимую в показание измерительного прибора, называют ***поправкой к показанию прибора***.

*Note*: The correction sign is opposite to the error sign. The correction added to a nominal measure value, is called the correction to the to the measure value; the correction introduced into the indication of measuring instrument, is called the correction to the indication of  instrument.

The correction can be presented in the form of an additional summand, i.e., the additive correction, and in the form of a multiplier, i.e., the multiplicative correction.

**Measurement repeatability** is the measurement precision under a set of repeatability conditions of measurement.

**Measurement precision** is the closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.

**Repeatability condition of measurement** is the condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time.

**Instability (of the metrological characteristic)** is the variation of the metrological characteristic of a measuring instrument within a given time interval.

3.2 Accepted abbreviations and symbols

In the recommendations given the following abbreviations and symbols are used:

MI – measuring instrument;

MT – measuring instrument or measurement transducer;

SD - (standard) deviation;

PD- probability distribution (of a random variable);

CI – recalibration interval;

VI – verification interval;

MC – metrological characteristic;

SSSRD –state system of standard reference data;

$X_{ref}, x_{ref}$ – Quantity and its value reproduced by a standard measure;

$X_{cal}, x_{cal}$ – Quantity and its value reproduced by a calibrated measure;

$y_{ref}\left(x_{ref}\right),\ y_{ref}\left(x_{cal}\right)$ – Indication of a standard measuring instrument corresponding to the values reproduced by standard and calibrated measures, correspondingly. In cases when indications of the measuring instrument has the dimension of a measurand, then instead of $y_{ref}$ ($x_{ref}$) it is possible to use the designation $x_{ref}$ ;

$y_{cal}\left(x_{ref}\right)$is the indications of a calibrated measuring instrument, which correspond to the values reproduced by a standard measure;

$f_{nominal}\left(x\right),\ f_{ref}\left(x\right),\ f_{cal}\left(x\right)$is the nominal calibration function, calibration functions of a standard and calibrated MI, correspondingly

$u_{rel}$ is the relative standard uncertainty.

## 4 General regulations

4.1 Aim and tasks of calibration

Calibration is the procedure of disseminating measurement unit reproduced and/or retained by a measurement standard to a standard that is less accurate or to a MI (further to a standard under calibration) by the way of determining the ratio between the quantity values obtained with the help of the standard, and corresponding indications of the MI under calibration.

Further, when using the MI as it is intended for, this is applied for transducing the MI indications into measurement results.

In calibration they determine metrological characteristics of measuring instruments.

4.2 Forms of presenting calibration characteristics

4.2.1 As metrological characteristics there can be measure values, error (systematic) of measuring instruments, calibration characteristic, deviations from nominal values of MI calibration characteristics and others. When performing the MI calibrations, the metrological characteristics are indicated with the corresponding uncertainties.

4.2.2 The value of a single-value measure, which is determined in calibration, is indicated as a new value or correction (additive or multiplicative) to the nominal value or to a value assigned to the measure in its preceding previous.

In calibration of a multivalued measure a whole group of new values or corrections are indicated for all range points under calibration.

4.2.3 The MI calibration characteristic indicated in form of table or function.

If MT indications have dimension of the measured value, than the most general procedure to present calibration characteristic is a task in form of table of consistent pairs of values measured $x_i$, $i = 1,...,n$ and indications of amendments to the MT. This case is considered in these recommendations.

4.2.4 In those cases when units of MT indications differ from a measurand the calibration characteristic is given by the parametrical functional dependence of the MT indications on the measurand values. In this case the MT calibration consists in evaluating the parameters of such function on the basis of values obtained with the help of the standard MI and corresponding indications of the MT being under calibration.

A particular case of the calibration function is the linear dependence going through the zero when a singular parameter of evaluation is the calibration coefficient $K (y = Kx)$. Taking into account a wide application of linear calibration functions of the type $y = a + bx$ in practice, their evaluation is also included in these recommendations.

4.2.5 The calibration characteristic can also be given by corrections (additive and/ or multiplicative) to an assigned (nominal) MT calibration characteristic.

4.2.6 In case of need in the process of calibration it is possible to determine, for example, such metrological characteristics of measuring means as:

- instability of a calibration characteristic of a measuring means;
- RMS of MT indications under the conditions of repeatability, which characterizes a random scattering of indications under normal conditions of calibration;
- nonlinearity of a calibration function.

4.3 Methods of measurements applied in calibration of measuring means measures.

4.3.1 Calibration of single-value and multivalued measures can be performed by the following methods:

- Method of direct measurements. According to this method the values of a measure under calibration are evaluated with the help of standard MT.

- Method of comparison with a standard measure with the help of a comparator. This method has two versions:
- Differential method of measurements, according to which the difference between the quantity values maintained stored by a calibrated and standard measures is evaluated.
- Method of substitution, according to which values of a calibrated and standard measures are successively determined with the help of MI used in the capacity of a comparator.
- Method of indirect measurements. This method makes it possible to determine the value of a measure on the basis of a known dependence of the quantity reproduced by the measure, on other quantities measured directly.

4.3.2 Calibration of MTs can be performed with the following methods:

- Method of direct measurements according to which the MT under calibration is used to measure values of a multiple-valued standard measure or a set of single-valued standard measures;
- Method of comparison with a standard MT. This method has two versions:

–Method of comparison with the help of a transfer standard (a multiple-valued measure or a set of single-valued measures).

–Method of direct comparison of a MT being under calibration with a standard MT.

- Method of indirect (joint or combined) measurements.

**5 Methods of evaluating a measurement result and its uncertainty in calibration of MIs.**

5.1 Order of evaluation.

Evaluation of a measurement result and its uncertainty is performed in the following order:

- formation of the measurement equation;
- evaluation of input quantities and their uncertainties;
- evaluation of output quantities and their uncertainties;
- formation of an uncertainty budget;
- presentation of calibration results.

5.2 Formation of a measurement equation in calibration.

5.2.1 In calibration the measurement equation expresses the dependence of a metrological characteristic of a MI (output quantity Y) which is determined, on all other quantities (input quantities $X_i$ where $i = 0,\ldots,n$) having an influence on getting an estimate of this metrological characteristic:

$$Y = F(X_0, X_1, ..., X_n)\,. \qquad (1)$$

*Note:* In calibration of multi-valued measures or MTs in some points of the scale, equation (1) is transformed into a system of equations:

$$\begin{cases} Y_1 = F_1(X_0, X_1, ..., X_n) \\ ---------------- \\ Y_m = F_m(X_0, X_1, ..., X_n) \end{cases}. \quad (2)$$

5.2.2 In calibration as an output quantity in the measurement equation there can be:

– value of a measure being calibrated or its deviation from a nominal value;
– systematic error of a given MT at a fixed point of the range;
– deviation of MT indications from the nominal calibration characteristic;
– calibration characteristic value at a point of the range;
– MT calibration coefficients;
– other MI metrological characteristics.

*Note:* If in calibration it is necessary to evaluate the stability of a measure, then a change of a measure value for a definite time interval equal to the difference of two results of measure values measurements, is taken as the output quantity.

5.2.3 In calibration, input quantities of the measurement equation are the quantities that influence a result of determining the MI metrological characteristic and its uncertainty, in particular:

$X_0$ – the quantity directly measured in calibration. Its value is determined / given with the help of a measurement standard being used in calibration. X can be the quantity at the input of the MT being calibrated or the quantity reproduce by the measure under calibration.

$X_1, ..., X_n$ – are the influencing quantities. Their values are measured either directly or they are the reference data, set constants and so on.

5.2.4 Forming the measurement equation it is necessary to take into account available information such as:

–nominal calibration function of the MI under calibration;
–calibration function of the standard MI;
–prior known type of the influence functions and corrections;
–any other information that permits the measurement equation be defined more accurately.

*Notes:* 1.The measurement equation is always some approximation of the dependence of the output quantity on the input ones the concrete form of which is determined by the requirements to the accuracy of determining the metrological characteristic in calibration. General recommendations will be given in 6.

2. Examples of measurement models applied at calibration will be given in consideration of measurement methods in 7 and 8.

5.3 Evaluation of input quantities and their standard uncertainties.

5.3.1 As an input quantity value its best estimate is taken.

5.3.2 Evaluation of the standard uncertainty according to Type A is applied when there are results *m* of independent measurements of one of the input quantities $X_i$ , $I = 0, \ldots, n$, performed under similar conditions: $x_{i1}, \ldots x_{in}$. As the value $x_i$ of this quantity the arithmetic mean is taken:

$$x_i = \bar{x}_i = \frac{1}{m}\sum_{j=1}^{m} x_{ij} \ .$$

The standard uncertainty is calculated by the formula of the RMS of an arithmetic mean value:

$$u(x_i) = u_A(x_i) = \sqrt{\frac{1}{m(m-1)}\sum_{j=1}^{m}(x_{ij} - x_i)^2} = \frac{S_i}{\sqrt{m}}, \text{ where } S_i = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}(x_{ij} - x_i)^2}. \quad (3)$$

5.3.3 If a number of independent measurements $m$ of the input quantity is small (less than 10), then it is recommended to use instead of expression (3) the following estimate of the standard uncertainty:

$$u(x_i) = u_A(x_i) = \sqrt{\frac{m-1}{m-3}} \times \frac{S_i}{\sqrt{m}}. \quad (4)$$

5.3.4 If a number of independent measurements, m, of the input quantity is small (less than 10) and the process of its measurement has been studied well and is under a statistical control, then the priori estimate of dispersion $\sigma_i$ (RMS of the repeatability) obtained as a result of processing a great number of previous measurements will be a more reliable estimate. In this case instead of (3) the following evaluation is recommended:

$$u(x_i) = u_A(x_i) = \frac{\sigma_i}{\sqrt{m}}.$$

5.3.5 Initial data for evaluating the value of a quantity and its standard uncertainty according to Type B are the following sources of priori information:

–data of the previous measurements of this quantity which are indicated in measurement protocols, evidences about calibrations or verifications, or other documents.

– norms of measurement accuracy indicated in technical documentation related to measurement methods and MIs;

–values of constants, reference data and their uncertainty;

– information about supposed distribution of quantity values contained in reports and literary sources;

– experience of a researcher or knowledge of general regularities which the properties of applied materials and MIs submit to.

5.3.6 There are cases of the Type B evaluation which differ from each other:

5.3.6.1 If only one value $x_i$ of the quantity $X_i$ is known, for example, the result of a single measurment, correction or reference datum, then such value is accepted as the estimate $x_i$. . Assessment of the standard uncertainty $u_B(x_i)$ is performed in the following way:

– if the estimate of the standard uncertainty $u(x_i)$ is known, then $u_B(x_i) = u(x_i)$;

– if the expanded uncertainty $U(x_i)$ and coverage factor $k$ are known, then the standard uncertainty is calculated by the formula:

$$u_B(x_i) = \frac{U(x_i)}{k}.$$

If the coverage factor is not indicated, then it is accepted that:

• k =1,73 , if there are grounds to assume, the equally likely distribution of possible values within the limits of U($x_i$) (for example, due to rounding of the measurement result);

• k = 2 , if there are grounds to assume that the normal distribution of possible values and that the estimate $U(x_i)$ corresponds to the coverage probability 0,95);

• $k$ = 2,6 , if there are grounds to assume that the normal distribution of possible values and that the estimate $U(x_i)$ corresponds to the coverage probability 0,99 (for example it has been obtained in attestation of primary and secondary measurement standards for which the confidence probability of 0,99 has been obtained);

- $k = 2$ in all other cases when information about probability distribution is absent.

If the estimate of the standard uncertainty is not known and it should be calculated on the basis of available prior information, or evaluated experimentally.

5.3.6.2 If it is possible to evaluate only the upper $a_+$ and lower $a_-$ limits of the $X_i$ quantity values (for example, the limit of the allowable MI error, field of temperature variation, error of rounding or truncating), then for its value a uniform distribution is accepted. In this case

$$x_i = \frac{1}{2} \cdot (a_+ + a_-), \qquad u_B(x_i) = \frac{a_+ - a_-}{2\sqrt{3}}.$$

If $a_+ = -a_- = a$, then $u_B(x_i) = \frac{a}{\sqrt{3}}$.

5.4 Evaluation of output quantities and their uncertainties.

5.4.1 In calibration the result of measurements is calculated by formula (1) or (2), inserting values of the input quantities.

5.4.2 The contribution of $u_i(y)$, where $I = 0,\ldots,n$ (or contributions $u_i(y_l)$ for each $l = 1,\ldots, L$ output signal) to the uncertainty of measurement $y$ (or $y_l$) of each input quantity $X_i$ . It is determined by the formula:

$$u_i(y) = |c_i| \cdot u(x_i),$$

where $c_i$ is the coefficient of sensitivity with respect to the input quantity $X_i$ expressing the degree of its influence on the change of the output quantity Y. It is equal to a particular $x_i$ derivative of the function $F(x_0, \ldots, x_n)$, calculated for input quantity values equal to their best estimates:

$$c_i = \frac{\partial F(x_0, \ldots, x_n)}{\partial x_i}, \quad (5)$$

5.4.3 Provided the measurement equation can not be written in the explicit form (1), at least relative to some input quantities, then the corresponding coefficients of sensitivity $c_i$ can be evaluated using the difference of input quantity values within the limits of the standard uncertainty by the formula:

$$c_i = \frac{y(x_0, \ldots, x_i + u(x_i), \ldots, x_n) - y(x_0, \ldots, x_i - u(x_i), \ldots, x_n)}{2u(x_i)}.$$

5.4.4 When the estimates of input quantities are not correlated the combined standard uncertainty of the output quantity is calculated by the formula:

$$u(y)=\sqrt{\sum_{i=0}^{n}u_i^2(y)}\,.$$

5.4.5 If the measurement equation constitutes the algebraic sum of uncorrelated summands, then each of them depends on one of the input quantities,

$$F(X_0, ..., X_n)=\sum_{i=0}^{n}\varphi_i(X_i)\,,$$

then the estimate of the output quantity is equal to:

$$y=\sum_{i=0}^{n}\varphi_i(x_i)\,,$$

and its combined standard uncertainty is as follows:

$$u(y)=\sqrt{\sum_{i=0}^{n}\left(\frac{\partial\phi_i(x_i)}{\partial x_i}\right)^2 u^2(x_i)}\,. \quad (6)$$

In a particular case at $\varphi_i(X_i)=p_i X_i$, where $i=0, ..., n$, formula (6) takes the following form:

$$u(y)=\sqrt{\sum_{i=0}^{n}p_i^2 u^2(x_i)}\,.$$

5.4.6 If the measurement equation constitutes the product of uncorrelated summands each of the last ones depends on one input quantity,

$$F(X_0, ..., X_n)=\prod_{i=0}^{n}\phi_i(X_i)\,,$$

then the output estimate is equal to

$$y=\prod_{i=0}^{n}\phi_i(x_i)\,,$$

and its combined standard uncertainty is the following:

$$u_{rel}(y)=\frac{u(y)}{y}=\sqrt{\sum_{i=0}^{n}\left(\partial\varphi_i(x_i)\Big/\partial x_i\right)^2\frac{u^2(x_i)}{\varphi_i^2(x_i)}}\,. \quad (7)$$

In a particular case at $\varphi_i(X_i)=X_i^{p_i}$, where $i=0, ..., n$ formula (7) takes the following form:

$$u_{rel}(y)=\sqrt{\sum_{i=0}^{n}p_i^2 u_{rel}^2(x_i)}\,.$$

5.4.7 If the estimates of input quantities are correlated, then the combined standard uncertainty of measurement result is calculated by the formula:

$$u(y)=\sqrt{\sum_{i=0}^{n}u_i^2(y)+\sum_{i,j=0,\, i\neq j}^{n}c_{ij}r(x_i,x_j)u(x_i)u(x_j)}\,, \quad (8)$$

where

$$c_{ij} = c_i \cdot c_j \,,$$

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} \,. \qquad (9)$$

Here *r(*$x_i$ *,* $x_j$ *)* is the correlation coefficient $x_i$ and $x_j$ , and *u(*$x_i$ *,* $x_j$ *)* is the covariance of the quantities $x_i$ and $x_j$.

It should keep in mind, in formula (8) summands of the second sum can be a negative.

5.4.8 Correlation of input quantities is evaluated in the following way.

5.4.8.1 Provided there are *n* pairs $\{x_{i5} ,\ x_{j5}\}^{n}_{i-1}$ of simultaneous repeated measurements of the quantities *xi* and *xj*. , then the covariation of these quantities is evaluated by the formula:

$$u(x_i, x_j) = \frac{1}{n(n-1)} \sum_{s=1}^{n} (x_{is} - \bar{x}_i)(x_{js} - \bar{x}_j) \,.$$

Then the correlation coefficient (factor) of these quantities is found with the help of formula (9).

5.4.8.2 If the similar measurement methods or reference data characterized by significant uncertainties are used for evaluating the input quantities, these quantities can be correlated. If the input quantities $X_1$ and $X_2$ depend on mutually independent variables $Q_l \ (l = 1, ..., L)$, their estimates $x_1 = g_1(q_1, q_2, ..., q_L)$ and $x_2 = g_2(q_1, q_2, ..., q_L)$ are correlated and the covariation of these estimates is equal to:

$$u(x_1, x_2) = \sum_{l=1}^{L} c_{1l} c_{2l} u^2(q_l) \,, \ (10)$$

where $c_{1l}, c_{2l}$ are the coefficients of sensitivity of the quantities $X_1$ and $X_2$ for variables values $Q_l \ (l = 1, ..., L)$ calculated by formula (5).

Since only the summands the sensitivity coefficients of which do not equal to zero make a contribution to sum (10), the covariation will be equal to zero, provided the functions $g_1$ and $g_2$ have no common variables.

5.4.8.3 If the correlation coefficients are not known, then the estimate over the combined standard uncertainty of measurement is given by the formula:

$$u^2(y) \le (|\, u_1(y)\,| + |\, u_2(y)\,|)^2 + u_r^2(y) \,,$$

where $u_r(y)$ is the contribution to the standard uncertainty of the measurand of the remaining input quantities that are considered as uncorrelated ones.

5.4.9 Correlation of two input quantities can be assumed as those that are equal to zero, or it is possible to consider it as an extremely small/ low correlation provided:

– these quantities are independent from each other (for example, if they were observed repeatedly, but not simultaneously in various experiments independently of one another) ;
– one of these quantities can be considered as a constant;
– there are no any reasons for correlation between these quantities.

Sometimes, the correlations can be excluded with the help of an appropriate choice of the measurement equation.

5.4.10 In (carrying out) calibration of measuring instruments it should estimate the correlation coefficients $r(y_q, y_{q+1})$, $q = 1, ..., Q-1$ between the estimates of output quantities $y_q$ $(q = 1, ..., Q)$ at adjacent points of the measurement range ($Q$ is the number of MI measurement range points). In the first approximation it is possible to calculate them, admitting that the systematic factors are invariable and consequently that the uncertainty components evaluated according to Type B are equal:

$$u_B(y_q) = u_B(y_{q+1}) = u_B(y). \quad (11)$$

Then the covariation of uncertainties of the quantities $y_q$, $y_{q+1}$ is equal to

$$u(y_q, y_{q+1}) = u_B(y_q, y_{q+1}) = u_B^2(y) \quad (12)$$

and the correlation coefficient (factor) of the quantities $y_q$, $y_{q+1}$, $q = 1, ..., Q-1$:

$$r(y_q, y_{q+1}) = \frac{u(y_q, y_{q+1})}{u(y_q)u(y_{q+1})} = \frac{u_B^2(y)}{u(y_q)u(y_{q+1})}. \quad (13)$$

5.4.11 If the standard uncertainty of MT indications according to Type B essentially depends on the measurand, it should be divided into two components one of which $u_{B1}(y)$ does not depend on the measurand, and the other depends on it linearly, $u_{B2}(y_q) = y_q u_{B2}$, then the calculations should be carried out by the formula:

$$u_B^2(y_q) = u_{B1}^2(y) + y_q^2 u_{B2}^2.$$

5.5 Working drawing up of the uncertainty budget.

5.5.1 Working drawing up of the uncertainty budget means a short formalized description of the procedure of measurement uncertainty evaluation. Such uniform scheme is descriptive**.** It allows the procedure of uncertainty calculation to be easily checked up, be compared with similar calculations in other laboratory.

5.5.2 Presentation of the uncertainty budget includes the description of the measurement equation and components of uncertainty in the form of a table (Table 1).

Table 1

Budget of uncertainty

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Quantity | Estimate | Standard uncertainty | Type of evaluation<br>Law of distribution | Sensitivity coefficient | Contribution to combined standard uncertainty |
| $X_i$ | $x_i$ | $u(x_i)$ | A (B) | $c_i = \frac{\partial F}{\partial x_i}$ | $u_i(y_q) = \left\lvert\frac{\partial F}{\partial x_i}\right\rvert \cdot u(x_i)$ |

| | | | | | |
|---|---|---|---|---|---|
| … | … | … | … | … | … |
| 1 | 2 | 3 | 4 | 5 | 6 |
| … | … | … | … | … | … |
| $Y_q$, $q = 1, ..., Q$ | $y_q = F(x_i)$ | $u(y_q)$ | | | $u(y_q) = \sqrt{\sum_{i=0}^{n} u_i^2(y_q)}$ Correlation coefficient (factor) $r(y_q, y_{q+1})$ |

In column 1 there are enumerated input qualities of the measurement equation.

In column 2 there are given estimates of input qualities obtained either as a result of measurements or on the basis of using other information.

In column 3 there are given values of the standard uncertainty of these estimates.

In column 4 there are indicated the type of uncertainty evaluation. In case of need supposed the normal law of distribution of its values and type of evaluation are supposed assumed. In case of repeated measurements it is also necessary to indicate the number *n* of measurements.

In column 5 there are sensitivity coefficients of input quantities $c_i = \frac{\partial F}{\partial x_i}$ .

In column 6 there are indicated values of input quantities contributions $u_i(y_q) = \left|\frac{\partial F}{\partial x_i}\right| \cdot u(x_i)$ to the combined standard uncertainty $u(y_q)$ (the product of the 3-rd column and module of the value from column 5).

In the last line of the Table there are given measurement results $y_q$ and their standard uncertainties $u(y_q)$. In this line for the MT the correlation coefficients $r(y_q, y_{q+1})$ between the output signals at the adjacent calibrated points of the measurement range are also indicated.

5.5.3 All quantities values shown in the Table have to include designations of units of these quantities. If the measurement uncertainty is given in a relative form, then this has to be indicated.

5.5.4 In some cases the uncertainty budget is worked out not for a concrete calibration point but for a range of measurand values or influencing quantities. In this case in columns 2, 3 and 6 the ranges of corresponding quantities changes are indicated.

5.6 Determination of expanded uncertainty

5.6.1 The expanded uncertainty $U(y)$ is equal to the product of the standard uncertainty $u(y)$ of the output quantity measurement result $y$ and the coverage factor $k$ :

$$U(y) = k\, u(y) .$$

5.6.2 The coverage factor depends on the type of output quantity distribution. In the general case it can obtained by statistical modelling at the known laws of input quantity distribution with the help of the Monte Carlo method.

5.6.3 If there are reasons to suppose that the measurement probability distribution is normal, the coverage factor is assumed to be equal to 2 ( $k = 2$ ). At that the expanded uncertainty of the measurement result approximately corresponds to the probability of coverage equal to 0,95.

5.6.4 In those cases when there is no information about the form of measurand uncertainty distribution, frequently with the purpose of unification it is recommended to consider that the coverage factor is equal to 2 ( $k = 2$ ) and that the expanded uncertainty of a measurement result will correspond to the probability of coverage of about 0,95.

5.6.5 The coverage factor equal to 2 is the estimate from the top in determination of the expanded uncertainty, provided two conditions are provided:

– there are two dominating sources of uncertainty: the accuracy of the measurement standard and random dispersion of measurement results in calibration;

– supposition of the effect of the normal law for describing the uncertainty and normal law for describing the dispersion of repeated measurement results. Expression (4) is used for the corresponding standard uncertainty according to Type A.

5.7 Presentation of calibration results.

5.7.1 Results of measure calibration can be presented by one of the methods listed below:

– by a value of a single value measure and corresponding expanded uncertainty with indication of the coverage factor;

– by deviation of the value of a single-value measure from the nominal value (or from the preceding value of calibration) as well as by the corresponding expanded uncertainty with indication of the coverage factor;

– for multi-value measures it is the deviations of real values from nominal values (or values of preceding calibrations) and corresponding expanded uncertainties with indication of the coverage factor;

– for multi-value measures it is a set of measure values and corresponding expanded uncertainties with indication of the coverage factor.

5.7.2 Results of MI calibration can be presented by one of the methods indicated below;

– Table of corrections to the nominal MT characteristic at each calibrated point of the measurement range , and corresponding expanded uncertainties with indication of the coverage factor;

– MT calibration coefficient and its expanded uncertainty with indication of the coverage factor;

– calibration function and expanded uncertainty at each point of the measurement range or parameters of the calibration function and uncertainties corresponding to them.

*Note.* For further application of the calibration characteristic it is useful to give the table of correlation coefficients of evaluated quantities $r(y_q, y_{q+1})$ at adjacent points of calibration.

5.7.3 For MTs and multi-valued measures the coverage factor $k$ is assumed to be the same for all measurement range points under calibration.

**6 Evaluation of measurement uncertainty constituents in calibration**

In this section there are given methods of evaluating typical constituents of the uncertainty, which are caused by:

- standards applied including:

– uncertainty of MI calibration characteristics and measure values;
– instability of MI calibration characteristics;
– nonlinearity of MI calibration characteristics

- random errors of measurement standards, calibrated means of measurements and calibration methods;
- method of measurements in calibration including an algorithm of calibration function parameters (the calculation of uncertainty for evaluating parameters of the linear calibration function by the least-squares method is given in Appendix A ).
- corrections caused by deviations from normal conditions.

6.1 Uncertainty of the calibration characteristic of standard MTs and values of standard measures applied in calibration.

6.1.1 Uncertainty constituent caused by this source is evaluated according to Type B. The information source is the calibration certificates or certificates if verification (further on for simplification – calibration certificates) of these measurement standards.

6.1.2 In calibration certificates it is necessary to indicate the expanded uncertainty $U$ (and coverage factor $k$ at which it was calculated) of the calibration characteristic:

- measure values when single valued and multi-valued measures are used as measurement standards;
- settings of zero and calibration coefficient when a MT with the linear calibration characteristic are applied as measurement standards;
- values of output signal indications at points $x_l$, $x_{l+1}$ of the standard MI measurement range which are the nearest ones in relation to the point under calibration $x$ , if the calibration characteristic of the standard MI is presented by a table of values.
- Values of output signal indications when the MI with a nonlinear calibration characteristic given in the form of a function is applied as the measurement standard.

*Note*. For measurement standards in the calibration certificate it is necessary to indicate the maximal value of the expanded uncertainty by the range or expanded uncertainty at points of calibration.

6.1.3 If in the verification certificate there are indicated confidential error limits $\pm\Delta_P$ corresponding to the confidence probability $P = 0{,}95$, then $U = \Delta_P$ and $k = 2$ are taken. If the confidence probability $P = 0{,}99$, then $U = \Delta_P$ and $k = 2{,}6$ are applied.

If in the verification certificate there is indicated a confidential error limit of the measurement standard characteristic $\Delta$, then $U = \Delta$ and $k = 3$ are applied.

6.1.4 The standard uncertainties are calculated by the formula:

$$u(x) = \frac{U}{k}.$$

6.1.5 When performing the calibration with the help of a standard MT, the uncertainty component at the calibrated point $x$ $(x_i < x < x_{i+1})$ caused by the uncertainty of the calibration characteristic of the standard MI in adjacent points $x_i, x_{i+1}$ is calculated by the formula:

$$u(y) = [u^2(y_i)(\frac{x_{i+1} - x}{x_{i+1} - x_i})^2 + 2r(y_i, y_{i+1})u(y_i)u(y_{i+1})\frac{(x_{i+1} - x)(x - x_i)}{(x_{i+1} - x_i)^2} +$$
$$+ u^2(y_{i+1})(\frac{x - x_i}{x_{i+1} - x_i})^2]^{0,5}.$$

Methods of determining the correlation coefficients are given in 5.4.7 – 5.4.11.

*Note*. If $u(y_i) = u(y_{i+1}) = u$ and the correlation coefficient is near the unit 1, then $u(y) = u$.

6.1.6 Provided the calibration characteristic of the standard MT is given analytically, in the form of a function of the values of calibration points *x*, then in the calibration certificate there are given standard uncertainties of parameters of this function and their correlation coefficients that allow the uncertainty caused by the calibration of the standard MT to be calculated, in particular:

- if the linear calibration characteristic goes through zero $y = kx$ and the standard uncertainty $u(k)$ of the calibration coefficient *k* is also given, then the corresponding uncertainty is equal to

$$u^2(y) = u^2(k)x^2;$$

- if the linear calibration characteristic does not go through zero: $y = a + bx$ and there are given standard uncertainties of a free (absolute) term $u(a)$, coefficient of the linear term $u(b)$, then the appropriate uncertainty is equal to

$$u^2(y) = u^2(a) + u^2(b)x^2 + 2xr(a,b)u(a)u(b).$$

*Note*. For the calibration characteristic given analytically in the form of a function, the value of the maximal uncertainty can be normalized reasoning from the measurement range in the absolute or relative form $u(y) < u_{\max}$.

6.2 Instability of measurement standards applied in calibration

6.2.1 Uncertainty of a calibration result caused by instability of the applied measurement standards is evaluated according to Type B. The information source is the minutes of calibrations and verifications performed during several years.

6.2.2 If the instability of measurement standards has a random character, then as a rule, it is normalized by the instability limits within the recalibration interval $\theta_{drift}$. The appropriate standard uncertainty is calculated by the formula:

$$u_{drift} = \frac{\theta_{drift}}{\sqrt{3}}. \quad (14)$$

*Note.* The instability of a standard MT can be normalized by the limits of time variation of the calibration coefficient $k$: $\theta_{drift}(k)$. The corresponding standard uncertainty of the calibration coefficient *k* is calculated by the formula:

$$u_{drift}(k) = \frac{\theta_{drift}(k)}{\sqrt{3}}. \quad (15)$$

If the instability of measurement standard has the systematic character, then in performing the calibration a correction is introduced:

$$\Delta t = \bar{v} \cdot t,$$

where $\bar{v}$ is the drift rate (drift speed), $t$ is the time run up from the moment of the last calibration.

The corresponding uncertainty is calculated by the formula:

$$u(\Delta t) = u(\bar{v}) \cdot t$$

Methods of calculating the uncertainty of the drift rate are given in Supplement B.

*Note.* If the instability of measurement standard is normalized by the limits depending on the time run up from the moment of the last calibration $\theta_{drift}(t)$, then the uncertainty component caused by measurement standard instability is calculated by formulae (14) and (15) with replacement of $\theta_{drift}$ by $\theta_{drift}(t)$ and $\theta_{drift}(k)$ by $\theta_{drift}(k,t)$, correspondingly.

6.3 Nonlinearity of the calibration function of a standard measuring instrument

6.3.1 Nonlinearity of the standard MT calibration function is determined at the stage of its calibration.

6.3.2 Correction introduced into MI indications at the point *x*, caused by the calibration function nonlinearity, is calculated by the formula:

$$\theta_{н}(x) = \frac{\sum_{i=1}^{m} y_i P_2(x_i)}{\sum_{i=1}^{m} P_2^2(x_i)} P_2(x),$$

where

$$P_2(x) = (x-\bar{x})^2 - (x-\bar{x})\frac{\sum_{i=1}^{m}(x_i-\bar{x})^3}{\sum_{i=1}^{m}(x_i-\bar{x})^2} - \frac{\sum_{i=1}^{m}(x_i-\bar{x})^2}{m}, \quad \bar{x} = \frac{1}{m}\sum_{i=1}^{m} x_i .$$

$\{x_i, y_i\}_1^m$ is the values of calibrated points and indications of the standard MI at these points, obtained in its calibration. Here $m$ is the number of calibrated point in calibration of the standard MI.

6.3.3 The correction uncertainty is calculated by the formula:

$$u(\theta_{н}) = \frac{u(y)}{\sum_{i=1}^{m} P_2^2(x_i)} |P_2(x)|,$$

where $u(y)$ the standard uncertainty is caused by repeatability of the standard MT indications.

6.3.4 If the nonlinearity correction is not introduced, then the corresponding uncertainty caused by nonlinearity of the calibration characteristic is evaluated from the top reasoning from the maximal deviation from the linear dependence by the formula:

$$u_н = \frac{\left| \frac{1}{4}\left( \frac{\sum_{i=1}^{m}\left(x_i - \overline{x}\right)^3}{\sum_{i=1}^{m}\left(x_i - \overline{x}\right)^2} \right)^2 + \frac{\sum_{i=1}^{m}\left(x_i - \overline{x}\right)^2}{m} \right|}{\sum_{i=1}^{m} P_2^2\left(x_i\right)} \cdot \sqrt{\frac{\left(\sum_{i=1}^{m} y_i P_2\left(x_i\right)\right)^2}{3} + u^2\left(y\right)} .$$

6.4 Random error of the measurement standard and MI under calibration

6.4.1 In case of the direct determination of a real value of the measure under calibration with the standard MT or when the standard MT is used as a comparator for comparisons, the random error of the standard MT becomes a component of the random error of a series of measurement results and the corresponding uncertainty can be evaluated according to Type A and 5.3.4. Provided the conditions of determining RMS of the standard MT repeatability coincide with the conditions of performing the calibration of the measure, then it is worthwhile to use a priori evaluation RMS of the repeatability in accordance with 5.3.4.

6.4.2 When calibrating a MT with the standard measure the random MT error is the component of the random error of determining the MT calibration characteristic and the corresponding uncertainty is evaluated according to Type A.

6.4.3 When the comparisons of the MT under calibration and the standard MT are realized directly, it is impossible, as a rule, to divide the random errors of these MTs and other components caused by variations of measurement conditions. The uncertainty caused by the combined random error is evaluated according to Type A in accordance with point 5.3.2 (Determination of the calibration characteristic).

6.4.4 In comparisons of calibrated and standard MT with the help of a reference measurement standard in calculation of the uncertainty caused by the random error of the standard MT, in some cases it is worthwhile to use priori estimates of repeatability of MT in accordance with 5.3.3.

6.5 Supplementary errors of measures in calibration

6.5.1 Supplementary errors of measures are called forth by the difference of measurement conditions and normal ones and to increase the measurement accuracy the corrections are introduced into them. The correction for the result of determining the MI metrological characteristic, ***y*,** that takes into account the $\alpha$ value of the influence quantity, is calculated by the formula:

$$\Delta y(\alpha) = c_0(y) \cdot (\alpha - \alpha_0),$$

where $c_0(y)$ is the nominal function of the quantity, $\alpha_0$ is the nominal value of the influence quantity.

6.5.2. The absolute standard uncertainty caused by the inaccuracy of this correction is calculated by the formula:

$$u[\Delta y(\alpha)] = \sqrt{u^2[c(y)] \cdot (\alpha - \alpha_0)^2 + u^2(\alpha) \cdot c_0^2(y)},$$

where $u[c(y)]$ is the standard uncertainty of the influence function value $\alpha$ .

6.5.3 The influence function values $c_l(y)$ are found from tables certified as SSRD or from other tables published by some competent metrological organization. The standard uncertainty or expanded uncertainty $U[c(y)]$ of these data and coverage factor $k$ should be taken from materials of organizations that have published these data.

6.5.4 The values $\alpha$ of influence quantities are determined by measuring these quantities. In this case the standards uncertainty is calculated in accordance with 5.3.

6.5.5 When the influence function or correction itself is presented in the form of a table, the correction calculation at an actual value of the influence quantity requires the linear interpolation between the nearest knots of interpolation $\alpha_l$, $\alpha_{l+1}$:

$$\Delta y(\alpha) = \Delta y(\alpha_l) + \frac{\Delta y(\alpha_{l+1}) - \Delta y(\alpha_l)}{\alpha_{l+1} - \alpha_l} \cdot (\alpha - \alpha_l).$$

6.5.6 The uncertainty of correction evaluation is formed from the estimate of the influence quantity, uncertainty of giving corrections (the influence function) in knots of interpolation and uncertainty caused by the linear interpolation:

$$u^2[\Delta y(\alpha)] = u^2[\Delta y(\alpha_{l+1})]\left(\frac{\alpha - \alpha_l}{\alpha_{l+1} - \alpha_l}\right)^2 + u^2[\Delta y(\alpha_l)]\left(\frac{\alpha_{l+1} - \alpha}{\alpha_{l+1} - \alpha_l}\right)^2 +$$
$$+ u^2(\alpha)\left(\frac{\Delta y(\alpha_{l+1}) - \Delta y(\alpha_l)}{\alpha_{l+1} - \alpha_1}\right)^2 + \frac{(\Delta y(\alpha_{l+2}) - 2\Delta y(\alpha_{l+1}) + \Delta y(\alpha_l))^2}{3}.$$

6.5.7 When the influence quantity correction $\alpha$ is not introduced, its estimate should be taken into account in calculation of the standard uncertainty of the MI metrological characteristic in the form:

$$u[y(\alpha)] = \sqrt{\Delta^2 y(\alpha) + u^2[\Delta y(\alpha)]}.$$

6.6 Rounding of measurement results

6.6.1 The uncertainty of a measurement result $y$ which is caused by its rounding (quantization) is evaluated in accordance with Type B. At that it is supposed that it is distributed according to the uniform law in a half of the width of first decimal range thrown off when rounding takes place.

6.6.2 The standard uncertainty caused by rounding is calculated by the formula:

$$u_{окр}(y) = \frac{0{,}5 \cdot 10^{-m(y)}}{\sqrt{3}} \cong 0{,}3 \cdot 10^{-m(y)},$$

where $m(y)$ is a serial number of the last significant digit of the measurement result $y$ (the scale-division value) of the MI applied for calibration).

6.6.3 In multiple measurements it is possible to consider the standard uncertainty of the measurement result as extremely small and if it does not exceed the standard uncertainty of this measurement evaluated in accordance with Type A (in cases when the position of the last significant digit of the measurement result does not exceed the position of the first significant digit of the expanded uncertainty according to Type A).

## 7 Calibration of measures

7.1 Calibration of measures with the direct measurement method

7.1.1. Calibration of a single-value measure consists in multiple measurements of the quantity reproduced by the measure under calibration, using the standard MI. Generally, the measurement equation is written in the form:

$$x_{cal} = f_{ref}^{-1}\left(y_{ref}\left(X_{cal}\right) + \sum \Delta y_i\right) + \sum \Delta x_i \ ,$$

where

$x_{cal}$ is the quantity value reproduced by the measure under calibration,

$f_{ref}$ , $f_{ref}^{-1}$ are the calibration functions of the standard and its inverse function,

$y_{ref}\left(X_{cal}\right)$ are the indication of the standard MI corresponding to the quantity reproduced by the measure under calibration,

$\Delta y_i$, $\Delta x_i$ are the corrections introduced into indications of the standard MI and final result, correspondingly.

7.1.2. A concrete form of the measurement equation depends on the method of presenting the calibration characteristic of the standard MI. Below there are given some typical methods of writing down the measurement equation.

7.1.3. If indications of the standard MI are presented directly in measurand units, then it corresponds to the identity nominal calibration function $f_{ref}(x) \equiv x$. In this case the calibration characteristic of the standard MI is presented by corrections for its indications and the measurement equation, as a rule, can be presented in the form:

$$x_{cal} = x_{ref} + \Delta x_{ref} + \sum \Delta x_i \ ,$$

where

$x_{ref}$ is the indication of the standard MI,

$\Delta x_{ref}$ is the correction for MI indications.

*Note*. By way of the estimate $x_{ref}$ a mean value of repeated indications of the standard MI is taken. The corresponding standard uncertainty is calculated in accordance with Type A. Generally the correction for the standard MI indication includes the correction caused by a systematic shift, nonlinearity and drift of the calibration characteristic of the standard MI and, as a rule, the corresponding uncertainties are calculated in accordance with Type B.

7.1.4. If the calibration characteristic of the standard MI is presented by the calibration coefficient $k$ , then the measurement equation, as a rule, can be presented in the form:

$$x_{cal} = \frac{y_{ref}\left(X_{cal}\right) + \sum \Delta y_i}{k} + \sum \Delta x_i \ .$$

*Notes:*

1. As $y_{ref}\left(X_{cal}\right)$, as a rule, a mean value of the repeated indications of the standard MI is taken. The corresponding standard uncertainty is calculated in accordance with Type B.

2. In the same way the case of two-parametric linear calibration dependence and dependence of an arbitrarily preset form are considered.

*Example (S11, [4]): Calibration of a calibrator of a temperature unit at the temperature of 180°C*

*When calibrating the measured temperature, which should be established in the measurement hole of the calibrator block temperature when the built-in temperature indicator shows 180°C. The temperature of under calibration hole is determinate by standard integrated platinum resistance thermometer in accordance with the following measurement equation:*

*In calibration*

$$t_X = t_S + \Delta t_S + \Delta t_D + \Delta t_{iX} + \Delta t_R + \Delta t_A + \Delta t_H + \Delta t_V$$

*where*

$t_S$ *is the value of temperature obtained by the measurement standard over (by) resistance measured with an a.c. bridge;*

$\Delta t_S$ *is the correction caused by the a.c. bridge;*

$\Delta t_D$, $\Delta t_{iX}$, $\Delta t_R$, $\Delta t_A$, $\Delta t_H$, $\Delta t_V$ *are the corrections caused accordingly by:*

– *drift of the measurement standard from the moment of its last calibration;*

– *final resolution of indications of the temperature unit calibrator;*

– *axial temperature nonuniformity (inhomogeneity) in a measurement opening (aperture, orifice);*

– *hysteresis;*

– *temperature oscillations in the process of measurement.*

7.2 Calibration of measures by the method of comparing with a standard measure. Differential method.

This method of calibration is realized with the help of two MIs: the standard measure with a nominal value equal to the nominal value of the measure under calibration, and MI applied as the comparator.

7.2.1 The differential measurement method consists in repeated measurements of the difference of quantity values maintained by the standard measure and measure under calibration. In the general case the measurement equation is written in the form:

$$x_{cal} = x_{ref} + f_{ref}^{-1}\left(y_{ref}\left(X_{cal} - X_{ref}\right) + \sum \Delta y_i\right) + \sum \Delta x_i \,,$$

where:

$x_{cal}$ is the value of a measure under calibration,

$x_{ref}$ is the value of the standard measure determined in its calibration

$y_{ref}\left(X_{cal} - X_{ref}\right)$ is the indications of the standard MI corresponding to the difference of quantities reproduced by the standard measure and measure under calibration,

$f_{ref}$, $f_{ref}^{-1}$ are the calibration function of the standard MI and the inverse function with respect to it,

$\Delta y_i$, $\Delta x_i$ are the corrections introduced into indications of the standard MI and final measurement result, correspondingly.

7.2.2 The concrete form of the measurement equation depends on the method of presenting the calibration characteristic of the standard MI. Below some typical methods of putting down measurement equations are listed.

7.2.3 If the indications of the standard comparator are presented directly in units of a measurand, then this corresponds to the identical calibration characteristic $f_{ref}(x) \equiv x$ . In this case the calibration characteristic of the standard MI is presented in the form of corrections to its indications, and the measurement equation, as a rule, can be presented in the form:

$$x_{cal} = x_{ref} + \Delta_{ref}\left(X_{cal} - X_{ref}\right) + \Delta\left(X_{cal} - X_{ref}\right) + \sum \Delta x_i \ ,$$

where

$\Delta_{ref}\left(X_{cal} - X_{ref}\right)$ – показания ИП, соответствующие разности величин, воспроизводимых калибруемой и эталонной мерами,

$\Delta_{ref}\left(X_{cal} - X_{ref}\right)$ is the MI indications corresponding to the difference of quantities reproduced by the standard measure and measure under calibration,

$\Delta\left(X_{cal} - X_{ref}\right)$ is the correction to MI indications.

*Notes.*

1. In the capacity $\Delta_{ref}\left(X_{cal} - X_{ref}\right)$, as a rule, the average value of indications of the standard comparator is taken. The corresponding standard uncertainty is calculated in accordance with Type A. The uncertainties of the rest input quantities , as a rule, are calculated in accordance with Type B.
2. In the same way the case of the two-parametric linear calibration dependence and dependence of the form given arbitrarily.

7.2.4 If the calibration characteristic of the standard comparator is presented by the calibration coefficient $k$ , then the measurement equation, as a rule, can be presented in the form:

$$x_{cal} = x_{ref} + \frac{\Delta_{ref}\left(X_{cal} - X_{ref}\right) + \sum \Delta y_i}{k} + \sum \Delta x_i \ .$$

7.2.5 A particular case of the differential method of measurements is the zero method according to which they try to achieve the equality of dimensions of the standard measure and measure under calibration. At the same time in the right-hand part of equations the member corresponding to indications of the standard comparator is equal to zero.

*Example (S4, [4]): Calibration of a plane-parallel end measure of nominal length is 50mm.*

*Calibration of the end measure of 50 mm is performed with the method of comparison with the help the comparator with the standard end measure of the same nominal length and material. The difference of middle lengths is determined at the vertical position of both end measures with the help of two indicators being in contact with upper and bottom surfaces. The* *length of the measure under calibration is determined in accordance with the following measurement equation:*

$$l_X = l_S + \Delta l_D + \Delta l + \Delta l_C + \Delta l_t + \Delta l_V$$

*where:*

$l_S$ *is the length of the standard end measure at the temperature of* $t_0 = 20\,^0C$ *according to the certificate of its calibration;*

$\Delta l_D$ *is the change of the standard end measure of length from the moment of its last calibration;*

$\Delta l$ *is the difference of lengths of the measure under calibration and the standard end measure;*

$\Delta l_C$, $\Delta l_C$, $\Delta l_V$ *are the correction caused by lack of coincidenc of axes of the comparator; temperature correction; correction caused by deviation of the middle length of the end measure under calibration with indicators being in contact with the upper and bottom measurement surfaces.*

7.3 Calibration of measures by the method of comparing with the standard measure. Method of substitution.

This method of calibration, as in point 7.2, anticipates application of two MIs, namely a standard measure with a nominal value equal to a nominal value of the measure under calibration and MI used as the comparator.

7.3.1 Method of substitution consists in repeated alternate measurements performed by a comparator of quantity dimensions maintained by the standard measure and measure under calibration. In the general case the measurement equation has the form:

$$x_{cal} = x_{ref} + f_{ref}^{-1}\left(y_{ref}\left(X_{cal}\right) + \sum \Delta y_i\right) - f_{ref}^{-1}\left(y_{ref}\left(X_{ref} + \sum \Delta y_i\right)\right) + \sum \Delta x_i ,$$

where

$x_{cal}$ is the value of the measure under calibration,

$x_{ref}$ is the value of the standard measure determined in the process of calibration,

$y_{ref}\left(X_{cal}\right)$, $y_{ref}\left(X_{ref}\right)$ are the indications of the standard MI corresponding to the quantities reproduced by the calibrated and standard measures,

$f_{ref}$, $f_{ref}^{-1}$ are the calibrated function of the standard MI and function inverse with respect to it,

$\Delta y_i$, $\Delta x_i$ are the corrections introduced into indications of the standard MI and into the final measurement result, correspondingly.

7.3.2 If indications of the comparator are presented in units that differ from units of the measurand, then frequently in the measurement equation the ratio of indications of the standard comparator, which correspond to successively determined values of the measure under calibration and standard measure:

$$x_{cal} = \left(x_{ref} + \sum \Delta x_i\right) \frac{y_{ref}\left(X_{cal}\right)}{y_{ref}\left(X_{ref}\right)} \cdot \prod \delta x_i ,$$

where

$\Delta x_i$ is the corrections introduced into the standard measure value,

$\delta x_i$ is the multiplicative corrections.

7.3.3 There are such situations when in calibration of two standard measures having nominal values close to the value of the measure under calibration $x_{ref1} \le x_{cal} \le x_{ref2}$, are used. In this case first of all the linear calibration characteristic of the comparator is determined within the narrow range using these measures, after that using the result of this determination the value of the measure under calibration is evaluated. The measurement equation has the form:

$$x_{cal} = \frac{\left(x_{ref1} y_{ref}\left(X_{ref2}\right) - x_{ref2} y_{ref}\left(X_{ref1}\right)\right) + y\left(X_{cal}\right) \cdot (x_{ref2} - x_{ref1})}{y_{ref}\left(X_{ref2}\right) - y_{ref}\left(X_{ref1}\right)} \prod \delta x_i \ ,$$

where

$y_{ref}\left(X_{ref1}\right),\ y_{ref}\left(X_{ref2}\right),\ y_{ref}\left(X_{cal}\right)$ are the indications of the standard MI which correspond to the values of the standard measure and measure under calibration.

*Note*. As a rule, the mean values of repeated indications of the standard MI are taken as $y_{ref}\left(X_{ref1}\right),\ y_{ref}\left(X_{ref2}\right),\ y_{ref}\left(X_{cal}\right)$ and the corresponding standard uncertainties are also as a rule are calculated in accordance with Type A. The uncertainties of the rest input quantities are calculated in accordance with Type B.

7.3.4 When using the substitution method, the instability corrections introduced into indications of the standard MI, nonlinearity of the calibration characteristic and other systematic effects are not, as a rule, introduced, since measurements are performed within a short period of time and for measures having nominal values close to each other.

*Example (S, [4]): Calibration of a weight with the nominal value of 10 kg.*

*Calibration of the weight with the nominal mass value of 10 kg is accomplished by the method of comparing with the standard weight of the same nominal value with the help of a comparator the metrological characteristics of which are known. A conventional mass of the weight under calibration is determined in accordance with the following measurement equation:*

$$m_X = m_S + \Delta m_D + \Delta m + \Delta m_C + \Delta B,$$

*where*

$m_S$ *is the conventional mass of the standard weight;*

$\Delta m_D$ *is the value of a standard weight drift after the last weight calibration;*

$\Delta m$ *is the observed difference between the mass values of the weight under calibration and standard weight;*

$\Delta m_C$ *are the eccentricity and magnetic influence corrections, correspondingly.*

7.4 Calculation of the uncertainty in carrying out the calibration of measures

7.4.1 Typical sources of uncertainty in calibration of measures:

- uncertainty of the calibration characteristic of the standard MI,
- instability of the calibration characteristic of the standard MI,

– nonlinearity of the calibration characteristic of the standard MI,
– random error of the standard MI,
– uncertainty of the standard measure values,
– instability of the standard measures,
– influence of random factors caused by the procedure of measurements, for example an error of mounting the standard measure and measure under calibration on the comparator,
– calculation of errors,
– rounding of the measurement results,
– interpolation of table data.

7.4.2 Methods of calculation of the standard uncertainty of measurement standards, influence quantity corrections and corresponding uncertainties are given in section 6. The procedure of evaluating the standard and expanded uncertainties and compilation of the budget uncertainty is described in section 5.

7.4.3 As a rule, it is possible not to take into account the contribution to the uncertainty, caused by the instability and nonlinearity of the calibration function of the standard comparator in the substitution method, since they have an influence on measurements of standard measures and measures under calibration in the same way. Moreover, it is required to take into account that frequently for measurement results the influence quantity corrections occur to be the correlated ones and in practice this fact significantly decreases the combined uncertainty of the value of the measure under calibration.

7.4.4 Determination of the result of multi-valued measure calibration and evaluation of the calibration result uncertainty is performed in the same way, subsequently for its each nominal value. In the recommendations given the case when additional restrictions are imposed on a multi-valued measure, e.g., the sum of angles of a polyhedral prism equal to 360 degrees, are not considered.

## 8 Calibration of measuring instruments

8.1 Calibration of MIs with the method of direct measurements

8.1.1 Calibration of MIs using the method of direct measurements consists in multiple measurements of quantities reproduced by the standard measures/calibrators corresponding to different scale readings of MI, which are carried out by the MI under calibration.

8.1.2 If in calibration of MIs it is necessary to determine the corrections that should be introduced into indications of this MI or the deviations from the nominal calibration characteristic at the point $x_{ref}$, then the measurement equation can be presented, as a rule, in the form:

- for the additive corrections

$$\Delta\left(x_{ref}\right) = -\left(y_{cal}\left(X_{ref}\right) - x_{ref}\right) + \sum \Delta x_i$$

or

$$\Delta\left(x_{ref}\right) = -\left(y_{cal}\left(X_{ref}\right) - f_{nominal}\left(x_{ref}\right)\right) + \sum \Delta x_i ,$$

- for the multiplicative corrections

$$\delta\left(X_{ref}\right)=\left(\frac{y_{cal}\left(X_{ref}\right)}{x_{ref}}\right)^{-1}\prod\delta x_i$$

or

$$\delta\left(X_{ref}\right)=\left(\frac{y_{cal}\left(X_{ref}\right)}{f_{nominal}\left(x_{ref}\right)}\right)^{-1}\prod\delta x_i\ ,$$

where

$y_{cal}\left(X_{ref}\right)$ is the indications of the MI under calibration at the point which correspond to the quantity reproduced by the standard measure $X_{ref}$ ,

$x_{ref}$ is the value of the standard measure,

$f_{nominal}\left(x_{ref}\right)$ is the value of the nominal calibration characteristic of MI at the point $x_{ref}$ ,

$\Delta x_i$, $\delta x_i$ are the instability corrections of the standard measure and other influence quantities.

*Note.* As a rule, the mean value (average value) of MI under calibration is taken as a rule in the capacity of the estimates $y_{cal}\left(X_{ref}\right)$ and the corresponding standard uncertainty is calculated in accordance with Type A. the uncertainties of the remaining input quantities is calculated in accordance with Type B.

8.1.3 If in MI calibration they determine its calibration coefficient *k*, then the measurement equation is represented in the form:

$$k=\frac{y_{cal}\left(X_{ref}\right)}{x_{ref}}\prod\delta x_i\ .$$

*Note.* Evaluation of linear dependence coefficients carried out with the least-squares method are given in Supplement (Appendix) A.

*Пример (S10, [4]): Calibration of a caliper*

*A steel caliper is calibrated with the help of standard end measures with the nominal length value within the range from 0,5 to 150 mm. In calibration they establish the deviation of caliper indication from the standard measure value (error) at nominal temperature* $t_0=20$ *in accordance with the following measurement equation:*

*Example (S10, [4]):*

$$E_X=l_{iX}-l_S+\Delta l_t+\Delta l_{iX}+\Delta l_M\ ,$$

*where*

$l_{iX}$ *is the caliper indication,*

$l_S$ *is the real value of the end measure length,*

$\Delta l_t$, $\Delta l_{iX}$, $\Delta l_M$ *are the corrections caused correspondingly by: difference of the temperature values of the caliper and end measure of length; final resolution of the caliper, mechanical effects such as an existing force Abbe errors, deviations from flatness and parallelism of measurement surfaces.*

8.2 Calibration of MIs with the method of comparison with the standard MI

8.2.1 Calibration of MIs with the method of comparison with the standard MI can be realized directly or with the help of the reference measurement standard (a set of measures).

8.2.2 When establishing the calibration characteristic of MI under calibration, first of all at each point of calibration the value of a measurand is determined by indications of the standard MI, i.e., by $y_{ref}\left(X\right)$, using its calibration characteristic $x_{ref} = f_{ref}^{-1}\left(y_{ref}\left(X\right)\right)$ and then the corresponding uncertainty is calculated. After that modelling and calculation of the uncertainty is reduced to calibration with the direct measurement method (point 8.1).

8.2.3 In this case the measurement equation is frequently divided into two equations in accordance with the listed above sequence of actions.

*Example (S5, [4]): Calibration of a thermocouple of Type N at the temperature 1000 ℃.*

*The thermocouples of Type N are calibrated by the way of comparison with two standard thermocouples of Type R in a horizontal furnace at the temperature of 1000 ℃. EMF generated by the thermocouples is measured with the help of a digital voltmeter, Measurement consists in two stages. That is why in this case the measurement model is given for each stage.*

*At the first stage they determine the temperature of a hot alloy using the standard thermocouples in the following way:*

$$\boldsymbol{t}_X = \boldsymbol{t}_S(\boldsymbol{V}_{iS} + \Delta\boldsymbol{V}_{iS}) + \Delta\boldsymbol{t}_D + \Delta\boldsymbol{t}_F\ ,$$

*where*

$t_S(V)$ *is the calibration function of the standard thermometer which allows the measured voltage value to be used for determining the temperature. The function is indicated in a calibration certificate;*

$\boldsymbol{V}_{iS}$ *is the voltmeter indication;*

$\Delta\boldsymbol{V}_{iS}$ *is the voltage value correction determined in voltmeter calibration;*

$\Delta\boldsymbol{t}_D$ *is the change of standard thermometer values from the moment of their last calibration due to a drift;*

$\Delta\boldsymbol{t}_F$ *is the temperature value correction caused by heterogeneity of the furnace temperature.*

*At the second stage the corresponding voltage VX arising in the thermocouple under calibration is determined:*

$$V_X(t) \cong V_X(t_X) + \frac{\Delta t}{C_X} - \frac{\Delta t_{0X}}{C_{X0}}\ ,$$

*where*

$V_X(t_X)$ *is the indication of the voltmeter;*

$\Delta t_{0X}$ *is the temperature value correction arising due to the deviation of the reference temperature from 0 ℃;*

$C_X$ *is the thermocouple sensitivity;*

*$C_{X0}$ is the thermocouple sensitivity with respect to voltage at the reference temperature 0 ℃;*

*t is the temperature at which the thermocouple has to be calibrated (the calibration point);*

*$\Delta t = t - t_X$ is the temperature deviation of the calibration point from the furnace temperature.*

8.3 Calculation of uncertainty in calibration

8.3.1 Typical uncertainty sources are:

– random errors of the standard MI and MI under calibration,
– uncertainty of the calibration characteristic of the standard MI,
– instability of the standard measure value,
– instability of the calibration characteristic of the standard MI,
– nonlinearity of the calibration characteristic of the standard MI
– rounding of measurement results,
– interpolation of table data
– heterogeneity of measurand distribution in the physical medium (the error of determining the measurand variation correction)
– difference of influence quantity values in measurements with the help of the standard MI and MI under calibration (the error of determining the corrections for these differences and influence of factors for which the corrections are not introduced).

8.3.2. Calculated dependences for estimates of standard uncertainty components of the calibration result, which are caused by such sources, are shown in section 6. The method of calculation of standard uncertainty and reporting uncertainty budget of calibration is given in section 5.

8.3.3 As a rule, when performing the calibration they obtain a set of repeated indications of the standard MI and MI under calibration, $y_{cal}\left(X_{ref}\right)$, $y_{ref}\left(X_{ref}\right)$. In this case in data processing it is necessary to take into account a possible correlation of indications of the standard MI and MI under calibration, which is caused by measurand fluctuations. In particular, may be it will be worthwhile to apply the reduction method.

## 9 Additional tasks being solved in calibration

In calibration of standard measures and MIs in case of a need it is possible to solve the following tasks:

- evaluation of the instability of measures and MI calibration characteristics,
- evaluation of RMS values of the indication repeatability of MI under calibration,
- evaluation of the nonlinearity of the MI calibration characteristics.

9.1 Evaluation of the measures instability

9.1.1 In many types of measurements for the standard and reference measures of the highest category a requirement is set, i.e., the limit of an allowed variation of the measure value per recalibration interval. When this limit is exceeded, a new class of precision is given to the measure. For such measures the instability evaluation at a recalibration interval has to be a calibration component.

9.1.2 Variation (change) of the measure value at the ***l***-th recalibration interval is evaluated by the formula

$$dx_l = x_{l+1} - x_l ,$$

where $x_{l+1}$, $x_l$ are the values of the measure under calibration at the moments of the $l+1$-th and $l$-th calibrations which are determined in accordance with section 7.

9.1.3. Estimates of the values of the measure under calibration are obtained on the basis of repeated measurements. The estimates can be strongly correlated due to usage of one and the same standard MI and method of measurements. If during the recalibration interval of the measure the calibration of the standard MI was not performed, then it is possible not to take into account the uncertainty of its calibration characteristic and nonlinearity. The instability of the calibration characteristic of the standard MI should be evaluated only for the recalibration interval of the measure under calibration.

9.1.4 In the same way the evaluation of the instability of the MI calibration characteristic is carried out. As used here, material 6.2 is useful.

9.2 Evaluation of repeatability of measuring instruments repeatability.

9.2.1 The number of MI (critical) metrological characteristics to be normalized includes confidence bounds of the random error of measurements. For the intermittent monitoring of this characteristic, it is recommended to include the evaluation of the indication repeatability into the program of calibration of such MIs.

9.2.2 Evaluation of the standard deviation of the repeatability of indications, $S_r$, and its uncertainty is carried out in accordance with Type A by the way of a statistical analysis of repeated independent indications of MI under calibration, which correspond to the value of the quantity being reproduced by a stable measure and which were performed under conditions of repeatability. The number ***m*** of measurements has to be no less than 20. Processing of measurement results is accomplished in the following order.

9.2.3 Evaluation of the average value and root-mean-square deviation of measurement results is carried out by the formulae:

$$\bar{y} = \frac{1}{m}\sum_{j=1}^{m} y_j , \quad S_r = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}(y_j - \bar{y})^2} .$$

9.2.4 The series of $y_1, ..., y_m$ is checked up for detecting outliers on the basis of Grabbs' criterion. For this purpose Grabbs' statistics are found for the maximum $z_m = \max y_j$ and minimal $z_1 = \min y_i$ measurement results:

$$G_m = \frac{z_m - \bar{y}}{S}, \quad G_1 = \frac{\bar{y} - z_1}{S} .$$

Fulfilment of the conditions $G_m \le G_m^*$ and $G_1 \ge G_1^*$ where $G_m^*, G_1^*$ are the threshold values with the significance level is 1%, is evidence of the outliers lack (the table of $G_m^*, G_1^*$ is given in [4]). In this case the obtained value $S_r$ is assumed to be an estimate of the root-mean-square value of MI indications under the conditions of repeatability.

If one of these inequalities is not fulfilled, then it signifies that the corresponding value ($z_m$ or $z_1$) is the outlier. In this case it is excluded and the remaining series consisting of $m-1$ measurement results are processed in accordance with 9.2.3 and 9.2.4.

9.2.5 The relative standard uncertainty of the RMS deviation of repeatability is approximately calculated by the formula:

$$u_{rel}(S_r) = \sqrt{\frac{1}{2m}}.$$

9.2.6 The relative expanded uncertainty of the RMS deviation of repeatability is calculated by determining the following values:

– probabilities for the given probability of coverage P

– $p_1 = \frac{1-P}{2}$, $p_2 = \frac{1+P}{2}$;

– using an inverse distribution function $\chi^2$ with ($m$ -1) dergees of freedom find are values $100p_1$ and $100p_2$, i.e., the percentage points of this distribution $z_1 = z[(\chi^2_{m-1}(p_1))^{-1}]$, $z_2 = z[(\chi^2_{m-1}(p_2))^{-1}]$, where $(\chi^2_{m-1}(p))^{-1}$ is the function that is inverse with regard to the distribution function $\chi^2$ with ($m$ -1) degrees of freedom ( $p = F_{\chi^2(m-1)}(z)$ );

– relative expanded uncertainty of RMS deviation of repeatability:

$$U_{rel}(S_r) \cong \frac{\sqrt{z_2} - \sqrt{z_1}}{2\sqrt{m-1}}.$$

9.3 Evaluation of calibration characteristic nonlinearity

Evaluation of calibration characteristic nonlinearity is carried out in accordance with 6.3.

## 10 Usage of calibration results

10.1 Discussion of issues relating to usage of calibration results oversteps the framework of the document given. However, taking into account the importance of the issue this section contains short information. The detailed consideration of this issue can be found in [3].

Calibration results of MIs can be used for:

– check-up of the compliance of metrological characteristics of the MI under calibration with established requirements;
– calculation of the instrumental measurement uncertainty;
– establishment of the metrological traceability series.

10.2 When checking up the compliance of metrological characteristics of the MI under calibration with the requirements established, it is necessary to take into account the uncertainty of setting the given metrological characteristic in calibration.

Example. As an example of using the measurement uncertainty at the check-up of the metrological characteristics compliance with the requirements established, the document “OIML R-111-1 Weights of classes E1, E2, F1, F2, M1, M1-2, M2, M2-3 and M3. Part 1: Metrological and technical requirements” is given. In [5], when referring a weight to a definite class, two conditions are checked up:

1) For a weight under calibration, the expanded uncertainty $U(m)$ at $k = 2$ of the conventional mass has to be at most one third of its limits of allowable error $\delta m$ for the corresponding class:

$$U(m) \leq 1/3.\ \delta m$$

2) For each weight the conventional mass $\boldsymbol{m}$, determined with the expanded uncertainty $U(m)$ has not to differ from the nominal value of the weight mass, $\boldsymbol{m_n}$, by more than a limit of the allowable (assumed, assumptive, hypothetical) error minus the expanded uncertainty:

$$m_n - (\delta m - U(m)) \leq m \leq m_n + (\delta m - U(m))$$

Other guidance for using the uncertainty in testing the compliance with the requirements established are possible too.

10.3 The instrumental uncertainty is a component of the measurement uncertainty, caused by a MI applied. The measurement uncertainty is always greater than the instrumental one, since additional factors connected with conditions of measurements or application of MIs which result in additional sources of the uncertainty, appear.

In the general case, the uncertainty of measurement indicated in a calibration certificate is not the instrumental component of the measurement uncertainty, but it is the uncertainty of establishing a metrological characteristic of MI under calibration. When calculating the instrumental uncertainty of measurement, it is necessary to analyze the results of calibration of MI used and to determine whether it is rightful and necessary to introduce corrections into measurement results on the basis of results of MI calibration.

Provided the results of calibration are presented in the form of deviations of MI under calibration from the reference values of a measurand, which are defined by a standard, then the instrumental uncertainty will depend on:

– whether the correction is introduced in each point of the scale;

– whether the approximating dependence is established for a correction subject to the value of the measurand within the measurement range;

– whether the correction is taken into account in the total uncertainty and others.

Example. As an example let us consider the recommendations on establishing the uncertainty of scales in the process of their usage, which are given in EURAMET/cg-18/v.02 Guidelines on the Calibration of Non-Automatic Weighing Instruments.

In calibration of balances they establish the deviation from the reference value / error of balance indication, $E(R)$, and corresponding standard uncertainty, $u(E(R))$. When calculating the combined standard uncertainty of a weighing result, $u(W)$**,** they additionally take into account the uncertainty components caused by the resolution of balances and repeatability of indications.

If the correction on the calibration results is not introduced, then instead of $u(E(R))$ they use $\sqrt{u^2\left(E(R)\right)+E^2(R)}$.

## 11 Bibliography


1. JCGM 200:2008 International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM), BIPM 2008.
2. ISO/IEC Guide 98-3:2008 Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)
3. ISO/IEC Guide 98-4:2012 Uncertainty of measurement – Part 4: Role of measurement uncertainty in conformity assessment
4. EA-4/02 Expression of the uncertainty of Measurements in Calibration.
5. OIML R-111-1 Weights of classes E1, E2, F1, F2, M1, M1-2, M2, M2-3 and M3. Part 1: Metrological and technical requirements
6. EURAMET/cg-18/v.02 Guidelines on the Calibration of Non-Automatic Weighing Instruments

**APPENDIX A**

**Uncertainty of the linear calibration curve obtained by the least-squares method.**

A.1 For constructing the linear calibration characteristic of MI, measurements of a MI response are made at calibration points of the range considered with the known values of $x_1, ..., x_j, ..., x_N$. At each point $x_j$ they carry out ***n*** repeated measurements. Thus the series $y_{jl}$ $(l = 1, ..., n)$ of MI indication values at calibration points $x_j$ is formed.

A.2 The calibration characteristic is given by the expression

$$y = D_0 + K(x - \bar{x}),$$

where

$$D_0 = \frac{1}{N}\sum_{j=1}^{N}\bar{y}_j = \frac{1}{Nn}\sum_{j=1}^{N}\sum_{l=1}^{n} y_{jl}, \qquad K = \frac{\sum_{j=1}^{N}\bar{y}_j(x_j - \bar{x})}{\sum_{j=1}^{N}(x_j - \bar{x})^2},$$

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j, \qquad \bar{y}_j = \frac{1}{n}\sum_{l=1}^{n} y_{jl}.$$

A.3 In evaluation of the calibration characteristic uncertainty $y(x)$ it is necessary to take into account the uncertainty of values of the calibration points $u(x_j)$ evaluated according to Type B and uncertainties $u(y_{jl})$ MI output signals, which are evaluated according to both Type A (as a result of random errors of MI) and Type B (as a result of additional measurement errors in calibration),

A.4 If values of the calibration points $x_j$, $j = 1, ..., N$ are independent ( $\mathrm{cov}(x_j, x_k) = 0$ ) =0 for any $j, k = 1,...N$ at $j \neq k$ ) , then the standard uncertainty caused by an error of the least-squares method, is calculated by the formula:

$$u_{мнк}(x) = \sqrt{[\frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{j=1}^{N}(x_j - \bar{x})^2}] \cdot [u^2(\bar{y}) + K^2 u_B^2(x)]},$$

where

$$u(\bar{y}) = \sqrt{u_A^2(\bar{y}) + u_B^2(\bar{y})}, \quad (A.1)$$

$$u_A(\bar{y}) = \sqrt{\frac{1}{(Nn-2)n}\sum_{j=1}^{N}\sum_{l=1}^{n}[y_{ij} - D_0 - K(x_j - \bar{x})]^2}, \qquad (A.2)$$

$u_B(\bar{y}), u_B(x)$ are the standard uncertainties of the $\bar{y}$, $x$ evaluation according to Type B.

At the same time the standard uncertainties of model parameters are equal to

$$u(D_0) = \sqrt{\frac{u^2(\bar{y}) + K^2 u_B^2(x)}{N}}, \qquad u(K) = \sqrt{\frac{u^2(\bar{y}) + K^2 u_B^2(x)}{\sum_{j=1}^{N}(x_j - \bar{x})^2}}. \quad \text{(A.3)}$$

A.5 If the quantities $x_j$, $j = 1, ..., N$ are correlated (weighed down by the systematic error) then the standard uncertainty is calculated by the formula:

$$u_{мнк}(x) = \sqrt{[\frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{j=1}^{N}(x_j - \bar{x})^2}] \cdot u^2(\bar{y}) + K^2 u_B^2(x)}\ .$$

A.6 In a particular case, in constructing a linear calibration dependence going through zero, N calibration points with the values of $x_1, ..., x_j, ..., x_N$ (it is possible to apply only one multi-valued measure) are usually used. At each point $x_j$ they carry out $n$ measurements of the output signal with the results $y_{jl}$ $(l = 1, ..., n)$.

The calibration dependence is expressed by the formula:

$$y = Kx,$$

where

$$K = \frac{\sum_{j=1}^{N} \bar{y}_j x_j}{\sum_{j=1}^{N} x_j^2}, \qquad \bar{y}_j = \frac{1}{n}\sum_{l=1}^{n} y_{jl}\ .$$

If the results of measurements $x_j$, $j = 1, ..., N$ are independent, then the standard uncertainty of the method is calculated by the formula:

$$u_{мнк}(x) = u(K) \cdot x$$

In which

$$u(K) = \sqrt{\frac{u^2(\bar{y}) + K^2 u_B^2(x)}{\sum_{j=1}^{N} x_j^2}}\ ,$$

$u(\bar{y})$ is calculated by the formula (A.1) , $u_A(\bar{y})$ by the formula (A.2) in which $D_0 = 0$ .

If the quantities are correlated, then the standard uncertainty is calculated by the formula (A.3) in which

$$u(K) = \sqrt{\frac{u^2(\bar{y})}{\sum_{j=1}^{N} x_j^2} + K^2 u_B^2(x) \frac{(\sum_{j=1}^{N} x_j)^2}{(\sum_{j=1}^{N} x_j^2)^2}}$$

**APPENDIX B**

**Calculation of the uncertainty of the drift speed of metrological characteristics of measurement standards.**

B.1 Statistic estimates of the calibration characteristic drift of a measurement standard, i.e., estimates of the average drift speeds $\bar{v}_1, ..., \bar{v}_r$ and standard uncertainties of drift speeds $u(\bar{v}_1), ..., u(\bar{v}_r)$ where *r* is the number of calibration points, are determined on the basis of calibrations carried out for some years.

B.2 Standard indication corrections are determined using the values of $\bar{v}_1, ..., \bar{v}_r$ in carrying out the calibration with their help and taking into account the values of $u(\bar{v}_1), ..., u(\bar{v}_r)$, i.e., standard uncertainties of these corrections.

B.3. If the standard is the one-valued measure, on the basis of calibration 1 or verifications protocols Table B.1 is filled in.

Table B.1

Determination of the speed drift of one-valued measure values

| Serial number of calibration | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Date of calibration | | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ |
| Recalibration interval | | – | $t_1 = T_2 - T_1$ | $t_2 = T_3 - T_2$ | $t_3 = T_4 - T_3$ | $t_4 = T_5 - T_4$ |
| Measure value, quantity unit | Prescibed value | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ |
| | Real value | $\dot{y}_1$ | $\dot{y}_2$ | $\dot{y}_3$ | $\dot{y}_4$ | $\dot{y}_5$ |
| Drift speed, quantity unit/ Unit of time | | – | $v_1 = \frac{\dot{y}_2 - y_2}{t_1}$ | $v_2 = \frac{\dot{y}_3 - y_3}{t_2}$ | $v_3 = \frac{\dot{y}_4 - y_4}{t_3}$ | $v_4 = \frac{\dot{y}_5 - y_5}{t_4}$ |

In this Table the drift speed of the measure within the ***l-th*** recalibration interval is calculated by the formula:

$$v_l = \frac{\dot{y}_{l+1} - y_{l+1}}{t_l} \quad \text{(B.1)}$$

where

$t_l$ is the duration of the ***l-th*** recalibration interval;

$y_l$ is the assigned value of the measure at its arrival for the ***l-th*** calibration of $y_l = \dot{y}_{l-1}$;

$\dot{y}_l$ is the real value of the measure determined at the ***l-th*** calibration.

B.4 Further the calculations are carried out in the following order:

- The average speed of the measure value drift is determined as follows:

$$\bar{v} = \frac{1}{L}\sum_{l=1}^{L} v_l \ , \quad \text{(B.2)}$$

where L is the number of recalibration time intervals expired up to the moment considered as well as the standard uncertainty (according to Type A) of the average drift speed:

$$u(\bar{v}) = \sqrt{\frac{1}{L(L-1)}\sum_{l=1}^{L}(v_l - \bar{v})^2}\ . \quad \text{(B.3)}$$

- Drift correction of the standard measure is assumed to be equal to $\Delta t = \bar{v} \cdot t$, where $t = T - T_L$ is the time expired after the last calibration of the standard measure;
- Standard uncertainty if this correction is determined as $u(\Delta t) = u(\bar{v}) \cdot t$ with the number of the degree of freedom $L-1$.

B.5 If the standard is the multi-valued measure, then the corrections $\Delta_i t$ for the real values and their standard uncertainties $u(\Delta_i t)$ are determined in accordance with B.4 and B.5 for each value of the multi-valued measure.

B.6 If the standard is the measuring instrument with the linear characteristic, then on the basis of calibration protocols or verifications Table B.2 is filled up.

Table B.2

Determination of the drift speed of the MI calibration coefficient with the linear calibration characteristic.

| Serial number of calibration | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Date of calibration | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ |
| МКИ, сутки Recalibration interval, day | – | $t_1 = T_2 - T_1$ | $t_2 = T_3 - T_2$ | $t_3 = T_4 - T_3$ | $t_4 = T_5 - T_4$ |
| Shift of zero before calibration | $x_{01}$ | $x_{02}$ | $x_{03}$ | $x_{04}$ | $x_{05}$ |
| Calibration coefficient | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ |
| | $\dot{K}_1$ | $\dot{K}_2$ | $\dot{K}_3$ | $\dot{K}_4$ | $\dot{K}_5$ |
| Drift speed of the additional error component | – | $v_{01} = \frac{x_{02}}{t_1}$ | $v_{02} = \frac{x_{03}}{t_2}$ | $v_{03} = \frac{x_{04}}{t_3}$ | $v_{04} = \frac{x_{05}}{t_4}$ |
| Drift speed of the calibration coefficient | – | $v_{K1} = \frac{\dot{K}_2 - K_2}{t_1}$ | $v_{K2} = \frac{\dot{K}_3 - K_3}{t_2}$ | $v_{K3} = \frac{\dot{K}_4 - K_4}{t_3}$ | $v_{K4} = \frac{\dot{K}_5 - K_5}{t_4}$ |

In this Table the drift speed of zero in the ***l-th*** recalibration interval is calculated by the formula:

$$v_{0l} = \frac{x_{0(l+1)}}{t_l}\ ,$$

where $x_{0(l+1)}$ is the shift of zero determined in the $(l+1)$-th calibration.

The average drift of the calibration coefficient is calculated by the formula:

$$v_{Kl} = \frac{\dot{K}_{l+1} - K_{l+1}}{t_l},$$

where $K_{l+1}, \dot{K}_{l+1}$ is the estimate of the calibration coefficient at arrival $(l+1)$ for calibration and after calibration.

Further the calculations are performed in the following order:

- average drift speeds of zero and calibration coefficient:

$$\bar{v}_0 = \frac{1}{L}\sum_{l=1}^{L} v_{0l}, \quad \bar{v}_K = \frac{1}{L}\sum_{l=1}^{L} v_{Kl},$$

and standard uncertainty (according to Type A) of these speeds:

$$u(\bar{v}_0) = \sqrt{\frac{1}{L(L-1)}\sum_{l=1}^{L}(v_{0l} - \bar{v}_0)^2}, \qquad u(\bar{v}_K) = \sqrt{\frac{1}{L(L-1)}\sum_{l=1}^{L}(v_{Kl} - \bar{v}_K)^2};$$

- correction caused by the drift of the MI characteristics is assumed to be equal to: $\Delta t = (\bar{v}_0 + \bar{v}_K \cdot x)\cdot t$, where $x$ is the value of the quantity measured in calibration;
- standard uncertainty of this correction is assumed to be equal to:

$$u(\Delta t) = \sqrt{u^2(\bar{v}_0) + u^2(\bar{v}_K)\cdot x^2}\cdot t.$$

B.7 If the standard is the measuring instrument with the nonlinear calibration characteristic, then the data concerning the instability of MIs , $\{y_{il}, y_{(i+1)l}\}_{l=1}^{L}$, at points of the range $x_i$ and $x_{i+1}$, which are close to the calibration point $x$, are taken from the protocols of calibrations and verifications, i.e., ($x_i < x < x_{i+1}$): $y_{il} = y_l(x_i)$. These data are entered in Table B,3. Designations are similar with regard to those that are used in B.4., B.5.

Table B.3

Determination of the characteristics of MIs with the nonlinear calibration characteristic

| Serial number of calibration | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Date of calibration | | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
| Recalibration interval | | – | $t_1 = T_2 - T_1$ | $t_2 = T_3 - T_2$ | $t_3 = T_4 - T_3$ |
| Values at calibration points of the MI range | Assigned values | $y_{i1}$ $y_{(i+1)1}$ | $y_{i2}$ $y_{(i+1)2}$ | $y_{i3}$ $y_{(i+1)3}$ | $y_{i4}$ $y_{(i+1)4}$ |
| | Real values | $\dot{y}_{i1}$ $\dot{y}_{(i+1)1}$ | $\dot{y}_{i2}$ $\dot{y}_{(i+1)2}$ | $\dot{y}_{i3}$ $\dot{y}_{(i+1)3}$ | $\dot{y}_{i4}$ $\dot{y}_{(i+1)4}$ |
| Speeds of the values drift at the calibration points of the range, $x_i, x_{i+1}$ of the standard MI | | – | $v_{i1} = v_1(x_i) = \frac{\dot{y}_{i2} - y_{i2}}{t_1}$ | $v_{i2} = v_2(x_i) = \frac{\dot{y}_{i3} - y_{i3}}{t_{31}}$ | $v_{i3} = v_3(x_i) = \frac{\dot{y}_{i4} - y_{i4}}{t_3}$ |

| | | | | |
|---|---|---|---|---|
| | | $v_{(i+1)1} = v_1(x_{i+1}) = = \frac{\dot{y}_{(i+1)2} - y_{(i+1)2}}{t_1}$ | $v_{(i+1)2} = v_2(x_{i+1}) = = \frac{\dot{y}_{(i+1)3} - y_{(i+1)3}}{t_2}$ | $v_{(i+1)3} = v_3(x_{i+1}) = = \frac{\dot{y}_{(i+1)4} - y_{(i+1)4}}{t_3}$ |

In this Table the drift speeds $v_{il}$ are calculated by the formula (B.1).

Further they carry out the calculations in the order indicated below and determine the following:

- average drift speeds $\overline{v}_i, \overline{v}_{i+1}$ with the help of the formula (B.2):
- standard uncertainties of the average drift speeds $u(\overline{v}_i)$ and $u(\overline{v}_{i+1})$ by the formula (B.3) and the coefficient of the drift speeds at the calibration points $x_i$ and $x_{i+1}$ by the formula:

$$r(\overline{v}_i,\ \overline{v}_{i+1}) = \frac{1}{u(\overline{v}_i)u(\overline{v}_{i+1})}\frac{1}{L(L-1)}\sum_{l=1}^{L}(v_{il} - \overline{v}_i)(v_{(i+1)l} - \overline{v}_{i+1})\ ;$$

- correction caused by the MI instability is assumed to be equal to :

$$\Delta t = \frac{\overline{v}_i(x_{i+1} - x) + \overline{v}_{i+1}(x - x_i)}{x_{i+1} - x_i}\cdot t\ ,$$

where $t$ is the time since the last calibration of measurement standard

The standard uncertainty of the correction $\Delta t$ is equal to:

$$u(\Delta\,\mathrm{t}) = \sqrt{u^2(\overline{v}_i)\cdot(\frac{x_{i+1} - x}{x_{i+1} - x_i})^2 + 2r(v_i, v_{i+1})u(\overline{v}_i)u(\overline{v}_{i+1})\cdot\frac{(x_{i+1} - x)(x - x_i)}{(x_{i+1} - x_i)^2} + u^2(\overline{v}_{i+1})\cdot(\frac{x - x_i}{x_{i+1} - x_i})^2}\cdot t\ .$$

*Note.* If $u(\overline{v}_i) = u(\overline{v}_{i+1}) = u$ and the correlation coefficient is close to 1, then $u(\Delta t) = u\cdot t$ .